Department of Mathematics & Philosophy of Engineering

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(1) (i) (a) $y' = \frac{xy^3}{\sqrt{1+x^2}}$ $\frac{dy}{dx} = \frac{xy^3}{\sqrt{1+x^2}}$ $\int \frac{dy}{y^3} = \int \frac{x\,dx}{\sqrt{1+x^2}} + c'$ $\int y^{-3} dy = \int d\sqrt{1+x^2} + c'$ $\frac{y^{-2}}{-2} = \sqrt{1+x^2} + c'$ $y^{-2} = -2\sqrt{1+x^2} + c$		10
(b) $y' = e^{y-t} \sec y$ $\frac{dy}{dt} = \frac{e^y \sec y}{e^t (1+t^2)^{-1}}$ $\int e^{-y} \cos y\, dy = \int e^{-t}(1+t^2)\, dt + c$ $I_1 = \int e^{-y} \cos y\, dy = e^{-y} \sin y + \int e^{-y} \sin y\, dy$ $= e^{-y} \sin y - e^{-y} \cos y - \int e^{-y} \cos y\, dy$ $2I_1 = e^{-y} \sin y - e^{-y} \cos y$ $I_1 = {}^1/_2 (e^{-y} \sin y - e^{-y} \cos y)$ $I_2 = \int t^2 e^{-t}\, dt = -t^2 e^{-t} + \int 2te^{-t}\, dt = -t^2 e^{-t} + 2\left[-te^{-t} + \int e^{-t}\, dt\right]$ $= -t^2 e^{-t} - 2te^{-t} - e^{-t} = -e^{-t}(t^2 + 2t + 1) = -e^{-t}(t+1)^2$ By I_1 and I_2 ${}^1/_2 (e^{-y} \sin y - e^{-y} \cos y) = -e^{-t} - e^{-t}(t+1)^2 = -e^{-t}(1 + (t+1)^2) + c$		10

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(c)

$$2xy^2 + 4 = 2(3 - x^2y)\frac{dy}{dx}$$

$$2(3 - x^2y)\frac{dy}{dx} - 2xy^2 - 4 = 0$$

$$\frac{\partial U}{\partial y}\frac{dy}{dx} + \frac{\partial U}{\partial x} = 0$$

$$\frac{\partial U}{\partial y} = 2(3 - x^2y) \qquad \frac{\partial U}{\partial x} = -2xy^2 - 4$$

$$\frac{\partial^2 U}{\partial x \partial y} = 2(0 - 2xy) = -4xy \qquad \frac{\partial^2 U}{\partial y \partial x} = -2x\,2y = -4xy \Longrightarrow \text{Exact}$$

$$\frac{\partial U}{\partial y} = 2(3 - x^2y)$$

$$U = \int 2(3 - x^2y)\,dy + c(x)$$

$$U = 6y - x^2y^2 + c(x)$$

$$\frac{\partial U}{\partial x} = -2xy^2 + c'(x) = -2xy^2 - 4 \Longrightarrow c'(x) = -4$$

$$c(x) = -4x + c$$

$$\therefore U = 6y - x^2y^2 - 4x + c = 0$$

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(d) $\frac{2ty}{t^2+1} - 2t - (2 - \ln(t^2+1))y' = 0$

$$(2 - \ln(t^2+1))y' = \frac{2ty}{t^2+1} - 2t$$

$$(2 - \ln(t^2+1))\frac{dy}{dt} - \frac{2ty}{t^2+1} + 2t = 0$$

$$\underbrace{\frac{\partial U}{\partial y}}_{g(t,y)}\frac{dy}{dt} + \underbrace{\frac{\partial U}{\partial t}}_{f(t,y)} = 0$$

$$\frac{\partial U}{\partial y} = (2 - \ln(t^2+1)) \qquad \frac{\partial U}{\partial t} = -\frac{2ty}{t^2+1} + 2t$$

$$\frac{\partial^2 U}{\partial t \partial y} = \frac{-1}{(t^2+1)}2t \qquad \frac{\partial^2 U}{\partial y \partial t} = \frac{-2t}{t^2+1} \Longrightarrow \text{Exact}\left(\frac{\partial}{\partial t}U(t,y)\right) = 0$$

$$\frac{\partial U}{\partial y} = (2 - \ln(t^2+1))$$

$$U = \int (2 - \ln(t^2+1))\,dy + c(t) = (2 - \ln(t^2+1))y + c(t)$$

$$\frac{\partial U}{\partial t} = y\left(0 - \frac{1}{(t^2+1)}2t\right) + c'(t) = 2t - \frac{2ty}{t^2+1}$$

$$-\frac{2ty}{t^2+1} + c'(t) = 2t - \frac{2ty}{t^2+1} \Longrightarrow c'(t) = 2t \Longrightarrow c(t) = {2t^2}/{2} + c = t^2 + c$$

$$\therefore U = (2 - \ln(t^2+1))y + t^2 + c = 0$$

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(e) $2(x+2y)dx+(y-x)dy=0$ $2(x+2y)+(y-x)\frac{dy}{dx}=0$ $\frac{dy}{dx}=\frac{-2(x+2y)}{(x+2y)}=\frac{f(x,y)}{g(x,y)}$ $\frac{f(\alpha x,\alpha y)}{g(\alpha x,\alpha y)}=\frac{-2(\alpha x+2\alpha y)}{(\alpha x+2\alpha y)}=\frac{-2\alpha(x+2y)}{\alpha(x+2y)}=\frac{-2(x+2y)}{(x+2y)}=\frac{f(x,y)}{g(x,y)}$; Homo Let $v={}^{y}/_{x}$ $\quad vx=y \quad v+x\frac{dv}{dx}=\frac{dy}{dx}$ $2(x+2vx)+(vx-x)\left(v+x\frac{dv}{dx}\right)=0$ $2x(1+2v)+xv(v-1)+x^2(v-1)\frac{dv}{dx}=0$ $x(v^2+3v+2)+x^2(v-1)\frac{dv}{dx}=0$ $\int\frac{(v-1)}{(v+1)(v+2)}dv=-\int\frac{dx}{x}+\ln c$ $-2\int\frac{dv}{(v+1)}+3\int\frac{dv}{(v+2)}=-\ln x+\ln c$ $-2\ln(v+1)+3\ln(v+2)=-\ln x+\ln c$ $\ln\frac{(v+2)^3}{(v+1)^2}=-\ln x+\ln c=ln\frac{c}{x}$ $\frac{(v+2)^3}{(v+1)^2}=\frac{c}{x}$ $x(v+2)^3=c(v+1)^2$ $x\left({}^{y}/_{x}+2\right)^3=c\left({}^{y}/_{x}+1\right)^2$		10

(f)

$(x^2 - y^2)dx + xy\,dy = 0$ 10

$$\frac{dy}{dx} = \frac{(y^2 - x^2)}{xy} = \frac{f(x,y)}{g(x,y)}$$

$\frac{f(\alpha x,\alpha y)}{g(\alpha x,\alpha y)} = \frac{\left((\alpha y)^2 - (\alpha x)^2\right)}{\alpha y \alpha x} = \frac{\alpha^2(y^2 - x^2)}{\alpha^2(xy)} = \frac{(y^2 - x^2)}{xy} = \frac{f(x,y)}{g(x,y)}$; Homo

Let $v = {}^{y}\!/_{x}$ $\quad vx = y \quad v + x\frac{dv}{dx} = \frac{dy}{dx}$

$$v + x\frac{dv}{dx} = \frac{x^2(1 - v^2)}{x^2 v}$$

$$x\frac{dv}{dx} = \frac{-1}{v}$$

$$\int v\,dv = -\int \frac{dx}{v} + c$$

$$\frac{v^2}{2} = -\ln v + c$$

$$\frac{\left({}^{y}\!/_{x}\right)^2}{2} = -\ln\left({}^{y}\!/_{x}\right) + c$$

(ii) (a) 10

$$\frac{dy}{dx} = \frac{x}{x + \sqrt{xy}} = \frac{f(x,y)}{g(x,y)}$$

$\frac{f(\alpha x,\alpha y)}{g(\alpha x,\alpha y)} = \frac{\alpha x}{\alpha x + \sqrt{\alpha x \alpha y}} = \frac{\alpha x}{\alpha(x + \sqrt{xy})} = \frac{x}{x + \sqrt{xy}} = \frac{f(x,y)}{g(x,y)}$; Homo

Let $v = {}^{y}\!/_{x}$ $\quad vx = y \quad v + x\frac{dv}{dx} = \frac{dy}{dx}$

$$v + x\frac{dv}{dx} = \frac{x}{x\left(1 + \sqrt{v}\right)} = \frac{1}{\left(1 + \sqrt{v}\right)}$$

$$\int \frac{1 + v^{1/2}}{1 - v - v^{3/2}}\,dv = \int \frac{1}{x}\,dx + c$$

$$-{}^{2}\!/_{3}\ln\left(1 - v - v^{3/2}\right) - {}^{1}\!/_{3}\,v = \ln x + c$$

$$-{}^{2}\!/_{3}\ln\left(1 - \left({}^{y}\!/_{x}\right) - \left({}^{y}\!/_{x}\right)^{3/2}\right) - {}^{1}\!/_{3}\left({}^{y}\!/_{x}\right) = \ln x + c$$

(b)

$\frac{dy}{dx} = \frac{x^4 + 3x^2y^2 + y^4}{x^3 y} = \frac{f(x,y)}{g(x,y)}$ 10

$\frac{f(\alpha x,\alpha y)}{g(\alpha x,\alpha y)} = \frac{(\alpha x)^4 + 3(\alpha x)^2(\alpha y)^2 + (\alpha y)^4}{(\alpha x)^3(\alpha y)} = \frac{\alpha^4\left(x^4 + 3x^2y^2 + y^4\right)}{\alpha^4(x^3 y)} = \frac{x^4 + 3x^2y^2 + y^4}{x^3 y} = \frac{f(x,y)}{g(x,y)}$; Homo

$$\frac{dy}{dx} = \frac{1 + 3\left({}^{y}\!/_{x}\right)^2 + \left({}^{y}\!/_{x}\right)^4}{\left({}^{y}\!/_{x}\right)}$$

Let $v = {}^{y}\!/_{x}$ $\quad vx = y \quad v + x\frac{dv}{dx} = \frac{dy}{dx}$

$v + x\frac{dv}{dx} = \frac{1+3v^2+v^4}{v}$ $\int \frac{v}{(v^2+1)^2}\,dv = \int \frac{1}{x}\,dx + \ln c$ (apply a substitution $\frac{1}{(v^2+1)^2} = t$) $\int \frac{v}{(v^2+1)^2}\,dv = -\frac{1}{4}\int t^{-1/2}\,dt = -\frac{1}{4}\frac{t^{1/2}}{(1/2)} = \frac{-1/2}{(v^2+1)}$ $\frac{-1/2}{(v^2+1)} = \ln xc$ $\frac{-1/2}{\left((y/x)^2+1\right)} = \ln xc \longrightarrow y^2 = -x^2\left(1+\frac{1}{\ln cx^2}\right)$		
(iii) (a) $dx - \frac{1}{y^2-6y+13}\,dy = 0$ $\int dx - \int \frac{1}{y^2-6y+13}\,dy + c = 0$ $x - \int \frac{1}{(y-3)^2+2^2}\,dy + c = 0$ $x - \frac{1}{2}\tan^{-1}\left(\frac{y-3}{2}\right) + c = 0$ $y = 3 + 2\tan 2x + c$		10
(b)$\int (x^2+1)\,dx + \int \frac{1}{y}\,dy = 0$ $\ln y = -\frac{x^3}{3} - x - c$ $y = e^{\left(-\frac{x^3}{3}-x+c\right)}$ Initial Condition: $y(-1) = 1$; $1 = e^{\left(\frac{1}{3}+1-c\right)} \Longrightarrow c = 4/3$ $\ln y + \frac{x^3}{3} + x + 4/3 = 0$ $y = e^{-\frac{1}{2}(x^2+3x+4)}$		10

(02) (i) (a) $\cos x\ y' + \sin x\ y = 2\cos^3 x \sin x - 1$ $\underbrace{\frac{\partial U}{\partial y}}_{g(x,y)}\frac{dy}{dx} + \underbrace{\frac{\partial U}{\partial x}}_{f(x,y)} = F(x)$ $g = \cos x \quad f = \sin x\ y$ $\frac{\left(\frac{\partial f}{\partial y} - \frac{\partial g}{\partial x}\right)}{g} = \frac{\sin x - (-\sin x)}{\cos x} = 2\tan x$ I.F. $= e^{\int 2\tan x\,dx} = \frac{1}{\cos^2 x}$	5	25

$$\frac{\cos x}{\cos^2 x}\frac{dy}{dx}+\frac{\sin x\, y}{\cos^2 x}=\frac{2\cos^3 x\sin x}{\cos^2 x}-\frac{1}{\cos^2 x}$$

$$\sec x\,\frac{dy}{dx}+\tan x\sec x\, y=\sin 2x-\sec^2 x$$

$$\underbrace{\frac{\partial U}{\partial y}}_{g(x,y)}\frac{dy}{dx}+\underbrace{\frac{\partial U}{\partial x}}_{f(x,y)}=F(x)$$

$\frac{\partial g}{\partial x}=\sec x\tan x$ $\qquad$ $\frac{\partial f}{\partial y}=\sec x\tan x$ $\qquad$; exact

$\frac{\partial U}{\partial x}=\tan x\sec x\, y$ 15

$$U=\int \tan x\sec x\, y\;dx+c(y)$$

$$U=y\sec x+c(y)$$

$\frac{\partial U}{\partial y}=\sec x+c'(y)=\sec x \Longrightarrow c'(y)=0 \Longrightarrow c(y)=c$

$\therefore U=y\sec x+c$

But $\frac{d}{dx}U(x,y)=\sin 2x-\sec^2 x$ and $U=-\frac{\cos 2x}{2}-\tan x+d$

$$\therefore y\sec x+\frac{\cos 2x}{2}+\tan x=k$$

$y(\pi/4)=3\sqrt{2}$ $\qquad$ $3\sqrt{2}\sec(\pi/4)+\frac{\cos 2(\pi/4)}{2}+\tan(\pi/4)=k \Longrightarrow k=7$

5

$$y\sec x+\frac{\cos 2x}{2}+\tan x=7$$

(b) $ty'-2y=t^5\sin 2t-t^3+4t^4$ 25

$y'-2\frac{y}{t}=t^4\sin 2t-t^2+4t^{43}$

$$\frac{\left(\frac{\partial f}{\partial y}-\frac{\partial g}{\partial t}\right)}{g}=\frac{(2/t-0)}{1}=\frac{2}{t}$$

I.F. $=e^{\int\frac{2}{t}dt}=t^{-2}$

5

$(t^{-2}y)'=t^2\sin 2t-1+4t$

$(t^{-2}y)=\int t^2\sin 2t\,dt+\int(-1+4t)\,dt+c$

$(t^{-2}y)=-\frac{1}{2}t^2\cos 2t+\frac{1}{2}t\sin 2t+\frac{1}{4}\cos 2t-t+2t^2+c$

$y(t)=-\frac{1}{2}t^4\cos 2t+\frac{1}{2}t^3\sin 2t+\frac{1}{4}t^2\cos 2t-t^3+2t^4+ct^2$ 15

$y(\pi)=\frac{3}{2}\pi^4$;

$\frac{3}{2}\pi^4=-\frac{1}{2}\pi^4+\frac{1}{4}\pi^2-\pi^3+2\pi^4+c\pi^2 \Longrightarrow c=\pi-\frac{1}{4}$

$y(t)=-\frac{1}{2}t^4\cos 2t+\frac{1}{2}t^3\sin 2t+\frac{1}{4}t^2\cos 2t-t^3+2t^4+\left(\pi-\frac{1}{4}\right)t^2$ 5

(ii)

(a) 10

$y^{(4)} + 2y^{(3)} + 3y^{(2)} + 2y^{(1)} + y = 0$

$D^4 + 2D^3 + 3D^2 + 2D^1 + 1 = 0$

$(D^2 + D + 1)^2 = 0$

$D = {-1}/_{2} \pm i\,{\sqrt{3}}/_{2}, {-1}/_{2} \pm i\,{\sqrt{3}}/_{2}$

$y = (c_1 + c_3x)e^{-\left(\frac{1}{2}\right)x} cox\left(\frac{\sqrt{3}}{2}x\right) + (c_2 + c_4x)e^{-\left(\frac{1}{2}\right)x} sin\left(\frac{\sqrt{3}}{2}x\right)$

(b)

$y^{(5)} - 5y^{(4)} - 50y^{(3)} + 250y^{(2)} + 625y^{(1)} - 3125y = 0$ 20

$D^5 - 5D^4 - 50D^3 + 250D^2 + 625D - 3125 = 0$

$(D-5)^3(D+5)^2 = 0$

$D = 5,5,5,-5,-5$

$y = e^{5x}(c_1 + c_2x + c_3x^2) + e^{-5x}(c_4 + c_5x)$

(c)

$y^{(6)} - 5y^{(4)} + 16y^{(3)} + 36y^{(2)} - 16y^{(1)} - 32y = 0$ 20

$D^6 - 5D^4 + 16D^3 + 36D^2 - 16D - 32 = 0$

$(D+2)^2(D-1)(D+1)(D^2 - 4D + 8) = 0$

$D = -2, -2, 1, -1, 2 + 2i, 2 - 2i$

$y = c_1e^{2x}\cos 2x + c_2e^{2x}\sin 2x + e^{-2x}(c_3 + c_4x) + c_5e^x + c_6e^{-x}$

03. (i)

(a) $f(x) = x^3 + 2x^2 - 3x - 1$ has three roots 15

$|x_1 - x_0| \le 0.0005$ $\quad x_0 = -0.27 \quad x_1 = -0.29$

$f(x_0) = -0.06388 \quad f(x_1) = 0.01381 \quad f(x_0) * f(x_1) = -0.000083 < 0$

One root between -0.27 and -0.29

x_0	x_1	$x_1 - x_0$	$\lvert x_1 - x_0\rvert$	x_2	$f(x_0)$	$f(x_2)$	$f(x_0)f(x_2)$	Range for x^*
-0.27000	-0.29000	-0.02000	0.02000	-0.28000	-0.06388	-0.02515	0.00160679	x_2 to x_1
-0.28000	-0.29000	-0.01000	0.01000	-0.28500	-0.02515	-0.00570	0.00014334	x_2 to x_1
-0.28500	-0.29000	-0.00500	0.00500	-0.28750	-0.00570	0.00405	-0.00002307	x_0 to x_2
-0.28500	-0.28750	-0.00250	0.00250	-0.28625	-0.00570	-0.00083	0.00000471	x_2 to x_1
-0.28625	-0.28750	-0.00125	0.00125	-0.28688	-0.00083	0.00161	-0.00000133	x_0 to x_2
-0.28625	-0.28688	-0.00062	0.00062	**-0.28656**	-0.00083	0.00039	-0.00000032	x_0 to x_2
-0.28625	-0.28656	-0.00031	**0.00031**	-0.28641				

$|x_1 - x_0| = 0.00031 < 0.0005$

Thus solution is <u>-0.28656</u>

(b)

$f(x) = 3x - e^x$

$|x_1 - x_0| \le 0.0005 \quad x_0 = 0.5 \quad x_1 = 0.75$

$f(x_0) = -0.14872 \quad f(x_1) = 0.13300 \quad f(x_0) * f(x_1) = -0.0197799 < 0$ 15

One root between 0.5 and 0.75

x_0	x_1	$x_1 - x_0$	$\lvert x_1 - x_0 \rvert$	x_2	$f(x_0)$	$f(x_2)$	$f(x_0)f(x_2)$	Range for x^*
0.50000	0.75000	0.25000	0.25000	0.62500	-0.14872	0.00675	-0.00100447	x_0 to x_2
0.50000	0.62500	0.12500	0.12500	0.56250	-0.14872	-0.06755	0.01004681	x_2 to x_1
0.56250	0.62500	0.06250	0.06250	0.59375	-0.06755	-0.02952	0.00199395	x_2 to x_1
0.59375	0.62500	0.03125	0.03125	0.60938	-0.02952	-0.01116	0.00032930	x_2 to x_1
0.60938	0.62500	0.01563	0.01563	0.61719	-0.01116	-0.00214	0.00002393	x_2 to x_1
0.61719	0.62500	0.00781	0.00781	0.62109	-0.00214	0.00232	-0.00000497	x_0 to x_2
0.61719	0.62109	0.00391	0.00391	0.61914	-0.00214	0.00009	-0.00000019	x_0 to x_2
0.61719	0.61914	0.00195	0.00195	0.61816	-0.00214	-0.00103	0.00000220	x_2 to x_1
0.61816	0.61914	0.00098	0.00098	**0.61865**	-0.00103	-0.00047	0.00000048	x_2 to x_1
0.61865	0.61914	0.00049	**0.00049**	0.61890				

$|x_1 - x_0| = 0.00049 < 0.0005$

Thus solution is 0.61865

(c) $f(x) = x^3 - 4x + 9$

$|x_1 - x_0| \leq 0.0005$ $\qquad x_0 = -3 \qquad x_1 = -2$ 15

$f(x_0) = -6.000 \qquad f(x_1) = 9.000 \qquad f(x_0) * f(x_1) = -54.000 < 0$

One root between -3 and -2

x_0	x_1	$x_1 - x_0$	$\lvert x_1 - x_0 \rvert$	x_2	$f(x_0)$	$f(x_2)$	$f(x_0)f(x_2)$	Range for x^*
-3.00000	-2.00000	1.00000	1.00000	-2.50000	-6.00000	3.37500	-20.25000000	x_0 to x_2
-3.00000	-2.50000	0.50000	0.50000	-2.75000	-6.00000	-0.79688	4.78125000	x_2 to x_1
-2.75000	-2.50000	0.25000	0.25000	-2.62500	-0.79688	1.41211	-1.12527466	x_0 to x_2
-2.75000	-2.62500	0.12500	0.12500	-2.68750	-0.79688	0.33911	-0.27022934	x_0 to x_2
-2.75000	-2.68750	0.06250	0.06250	-2.71875	-0.79688	-0.22092	0.17604303	x_2 to x_1
-2.71875	-2.68750	0.03125	0.03125	-2.70313	-0.22092	0.06108	-0.01349296	x_0 to x_2
-2.71875	-2.70313	0.01563	0.01563	-2.71094	-0.22092	-0.07942	0.01754597	x_2 to x_1
-2.71094	-2.70313	0.00781	0.00781	-2.70703	-0.07942	-0.00905	0.00071872	x_2 to x_1
-2.70703	-2.70313	0.00391	0.00391	-2.70508	-0.00905	0.02604	-0.00023569	x_0 to x_2
-2.70703	-2.70508	0.00195	0.00195	-2.70605	-0.00905	0.00851	-0.00007697	x_0 to x_2
-2.70703	-2.70605	0.00098	0.00098	**-2.70654**	-0.00905	-0.00027	0.00000244	x_2 to x_1
-2.70654	-2.70605	0.00049	**0.00049**	-2.70630				

$|x_1 - x_0| = 0.00049 < 0.0005$

Thus solution is -2.70654

(ii)

(a) $f(x) = e^x \sin x - 1 \qquad f'(x) = e^x \cos x + e^x \sin x$

Method 01

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$x_{k+1} = x_k - \frac{(e^{x_k} \sin x_k - 1)}{e^{x_k} \cos x_k + e^{x_k} \sin x_k}$$ 10

$$x_{k+1} = \frac{e^{x_k}((x_k \cos x_k) + ((x_k - 1)\sin x_k)) + 1}{e^{x_k}(\cos x_k + \sin x_k)}$$

k	x_k	x_{k+1}
0	0.000	1.000
1	1.000	0.657
2	0.657	0.591
3	0.591	0.589
4	0.589	0.589
5	0.589	0.589

(b) $f(x) = x[1 - \log_e x] - 0.5$ $\quad f'(x) = x\left(0 - {}^{1}/_{x}\right) + [1 - \log_e x] = -\log_e x$ 10

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$x_{k+1} = x_k - \frac{(x_k[1 - \log_e x_k] - 0.5\,)}{-\log_e x_k}$$

$$x_{k+1} = \frac{x_k \log_e x_k + (x_k[1 - \log_e x_k] - 0.5\,)}{\log_e x_k}$$

$$x_{k+1} = \frac{x_k - 0.5}{\log_e x_k}$$

k	x_k	x_{k+1}
0.00000	0.10000	0.17372
1.00000	0.17372	0.18641
2.00000	0.18641	0.18668
3.00000	0.18668	0.18668

(c) $f(x) = x \sin x + \cos x$ $\quad f'(x) = x \cos x$ 10

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$x_{k+1} = x_k - \frac{(x_k \sin x_k + \cos x_k)}{x_k \cos x_k}$$

$$x_{k+1} = \frac{{x_k}^2 \cos x_k - (x_k \sin x_k + \cos x_k)}{x_k \cos x_k}$$

$$x_{k+1} = x_k - \tan x_k - \frac{1}{x_k}$$

k	x_k	x_{k+1}
0	3.142	2.823
1	2.823	2.799
2	2.799	2.798
3	2.798	2.798

(iii)

(a) $\int_0^5 \frac{dx}{4x+5}$ 10 equal parts $h = (5-0)/10 = 0.5$ $f(x) = \frac{1}{4x+5}$ 15

k	x	f
0	0	1/5
1	0.5	1/7
2	1	1/9
3	1.5	1/11
4	2	1/13
5	2.5	1/15
6	3	1/17
7	3.5	1/19
8	4	1/21
9	4.5	1/23
10	5	1/25

$$\int_0^5 \frac{dx}{4x+5} = \frac{0.5}{3}\left[\frac{1}{5}+\frac{1}{25}+4\left(\frac{1}{7}+\frac{1}{11}+\frac{1}{15}+\frac{1}{19}+\frac{1}{23}+2\left(\frac{1}{9}+\frac{1}{13}+\frac{1}{17}+\frac{1}{21}\right)\right)\right]$$
$=0.4025$

(b) $X = \int_8^{30}\left(2000\ln\left[\frac{140\ 000}{140\ 000-2100\ t}\right]-9.8t\right)dt$ 4 equal parts $h = (30-8)/4 = 5.5$ 10

k	x	f
0	8	177.2667
1	13.5	320.2469
2	19	484.7455
3	24.5	676.0501
4	30	901.6740

$$\int_8^{30}\left(2000\ln\left[\frac{140\ 000}{140\ 000-2100\ t}\right]-9.8t\right)dt$$
$$= \frac{5.5}{3}[177.2667+901.6740+4(320.2469+676.0501)+2(484.7455)]$$
$$= 11061.64$$

(04)(i) 40

Year x	1891	1901	1911	1921	1931
Population y	46	66	81	93	101

x	y	Δy_k	$\Delta^2 y_k$	$\Delta^3 y_k$	$\Delta^4 y_k$
1891	46	20	-5	2	-3
1901	66	15	-3	-1	
1911	81	12	-4		
1921	93	8			
1931	101				

10

1895: h=10 x0=1891 $x-1891=10s$ x=1895 $s=\frac{1895-1891}{10}=0.4$ 15

$$y_s=46+20s-5\binom{s}{2}+2\binom{s}{3}-3\binom{s}{4}$$

$$\binom{s}{2}=\binom{0.4}{2}=\frac{0.4(0.4-1)}{2!}=\frac{0.4\times(-0.6)}{2}=-0.12$$

$$\binom{s}{3}=\binom{0.4}{3}=\frac{0.4(0.4-1)(0.4-2)}{3!}=\frac{0.4\times(-0.6)\times(-1.6)}{6}=0.064$$

$$\binom{s}{4}=\binom{0.4}{4}=\frac{0.4(0.4-1)(0.4-2)(0.4-3)}{4!}=\frac{0.4\times(-0.6)\times(-1.6)\times(-2.6)}{4!}=-0.0416$$

$$y_s=46+20\times0.4-5\times-0.12+2\times0.064-3\times-0.0416=54.85$$

1925: h=10 x0=1931 $x-1931=10s$ x=1925 $s=\frac{1925-1931}{10}=-0.6$ 15

$$y_s=101-8s-4\binom{s}{2}+1\binom{s}{3}-3\binom{s}{4}$$

$$\binom{s}{2}=\binom{0.6}{2}=\frac{0.6(0.6-1)}{2!}=\frac{0.6\times(-0.4)}{2}=-0.12$$

$$\binom{s}{3}=\binom{0.6}{3}=\frac{0.6(0.6-1)(0.6-2)}{3!}=\frac{0.6\times(-0.4)\times(-1.4)}{6}=0.056$$

$$\binom{s}{4}=\binom{0.6}{4}=\frac{0.6(0.6-1)(0.6-2)(0.6-3)}{4!}=\frac{0.6\times(-0.4)\times(-1.4)\times(-2.4)}{4!}=-0.0336$$

$$y_s=101-8\times0.6-4\times-0.12+1\times0.056-3\times-0.0336=96.84$$

(ii)

(a) 30

x	300	304	305	307
Log x	2.4771	2.4829	2.4843	2.4871

$$Y=y_0\frac{(x-x1)(x-x2)(x-x3)}{(x0-x1)(x0-x2)(x0-x3)}+y_1\frac{(x-x0)(x-x2)(x-x3)}{(x1-x0)(x1-x2)(x1-x3)}+y_2\frac{(x-x0)(x-x1)(x-x3)}{(x2-x0)(x2-x1)(x2-x3)}+y_3\frac{(x-x0)(x-x1)(x-x2)}{(x3-x0)(x3-x1)(x3-x2)}$$

$$Y=2.4771\frac{(x-304)(x-305)(x-307)}{(300-304)(300-305)(300-307)}+2.4829\frac{(x-300)(x-305)(x-307)}{(304-300)(304-305)(304-307)}+2.4843\frac{(x-300)(x-304)(x-307)}{(305-300)(305-304)(305-307)}+2.4871\frac{(x-300)(x-304)(x-305)}{(307-300)(307-304)(307-305)}$$

X=310

$$Y=2.4771\frac{(310-304)(310-305)(310-307)}{(300-304)(300-305)(300-307)}+2.4829\frac{(310-300)(310-305)(310-307)}{(304-300)(304-305)(304-307)}+2.4843\frac{(310-300)(310-304)(310-307)}{(305-300)(305-304)(305-307)}+2.4871\frac{(310-300)(310-304)(310-305)}{(307-300)(307-304)(307-305)}$$

$$Y=2.4771\frac{(6)(5)(3)}{(-4)(-5)(-7)}+2.4829\frac{(10)(5)(3)}{(4)(-1)(-3)}+2.4843\frac{(10)(6)(3)}{(5)(1)(-2)}+2.4871\frac{(10)(6)(5)}{(7)(3)(2)}$$

$$Y=2.4771\frac{90}{-140}+2.4829\frac{150}{12}+2.4843\frac{180}{-10}+2.4871\frac{300}{42}=2.4914$$

(b)		
		30

x	1	2	4	7	8
U	22	30	82	106	206

$$u_6 = 22\frac{(6-2)(6-4)(6-7)(6-8)}{(1-2)(1-4)(1-7)(1-8)} + 30\frac{(6-1)(6-4)(6-7)(6-8)}{(2-1)(2-4)(2-7)(2-8)} + 82\frac{(6-1)(6-2)(6-7)(6-8)}{(4-1)(4-2)(4-7)(4-8)} + 106\frac{(6-1)(6-2)(6-4)(6-8)}{(7-1)(7-2)(7-4)(7-8)} + 206\frac{(6-1)(6-2)(6-4)(6-7)}{(8-1)(8-2)(8-4)(8-7)}$$

$$u_6 = 22\frac{(4)(2)(-1)(-2)}{(-1)(-3)(-6)(-7)} + 30\frac{(5)(2)(-1)(-2)}{(1)(-2)(-5)(-6)} + 82\frac{(5)(4)(-1)(-2)}{(3)(2)(-3)(-4)} + 106\frac{(5)(4)(2)(-2)}{(6)(5)(3)(-1)} + 206\frac{(5)(4)(2)(-1)}{(7)(6)(4)(1)}$$

$$u_6 = 22\frac{16}{126} + 30\frac{20}{-60} + 82\frac{40}{36} + 106\frac{-80}{-90} - 206\frac{40}{168} = 83.52$$

(05) — 35

(a) $5x_1 - 2x_2 + 3x_3 = -1$

$-3x_1 + 9x_2 + x_3 = 2$

$2x_1 - x_2 - 7x_3 = 3$

$x_1^{(0)} = x_2^{(0)} = x_3^{(0)} = 0$

$5x_1 - 2x_2 + 3x_3 = -1 \quad \rightarrow \quad x_1 = {(2x_2 - 3x_3 - 1)}/{5}$

$-3x_1 + 9x_2 + x_3 = 2 \quad \rightarrow \quad x_2 = {(3x_1 - x_3 + 2)}/{9}$ — 15

$2x_1 - x_2 - 7x_3 = 3 \quad \rightarrow \quad x_3 = {(2x_1 - x_2 - 3)}/{7}$

	x1	x2	x3
0	0.000	0.000	0.000
1	-0.200	0.222	-0.429
2	0.146	0.203	-0.517
3	0.192	0.328	-0.416
4	0.181	0.332	-0.421
5	0.185	0.329	-0.424
6	0.186	0.331	-0.423
7	0.186	0.331	-0.423
8	0.186	0.331	-0.423

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(b)

$x_1^{(0)} = x_2^{(0)} = x_3^{(0)} = 0$

$5x_1 - 2x_2 + 3x_3 = -1 \quad \rightarrow \quad x_1 = {(2x_2 - 3x_3 - 1)}/{5}$

$-3x_1 + 9x_2 + x_3 = 2 \quad \rightarrow \quad x_2 = {(3x_1 - x_3 + 2)}/{9}$

$2x_1 - x_2 - 7x_3 = 3 \quad \rightarrow \quad x_3 = {(2x_1 - x_2 - 3)}/{7}$

	x1	x2	x3
0	0.000	0.000	0.000
1	-0.200	0.156	-0.508
2	0.167	0.334	-0.429
3	0.191	0.333	-0.422
4	0.186	0.331	-0.423
5	0.186	0.331	-0.423
6	0.186	0.331	-0.423

20

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(c)

$2x_1 + x_2 + x_3 = 5$
$4x_1 - 6x_2 = -2$
$-2x_1 + 7x_2 + 2x_3 = 9$

$$\begin{bmatrix} 2 & 1 & 1 \\ 4 & -6 & 0 \\ -2 & 7 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ -2 \\ 9 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 4 & -6 & 0 \\ 2 & 1 & 1 \\ -2 & 7 & 2 \end{bmatrix}\begin{bmatrix} -2 \\ 5 \\ 9 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_1/4} \begin{bmatrix} 1 & -6/4 & 0 \\ 2 & 1 & 1 \\ -2 & 7 & 2 \end{bmatrix}\begin{bmatrix} -2/4 \\ 5 \\ 9 \end{bmatrix}$$

$$\xrightarrow[R_3 \leftrightarrow R_3 + 2R_1]{R_2 \leftrightarrow R_2 - 2R_1} \begin{bmatrix} 1 & -6/4 & 0 \\ 0 & 4 & 1 \\ 0 & -4 & 2 \end{bmatrix}\begin{bmatrix} -2/4 \\ 1 \\ 8 \end{bmatrix}$$

10

$$\begin{bmatrix} 1 & -6/4 & 0 \\ 0 & 4 & 1 \\ 0 & -4 & 2 \end{bmatrix}\begin{bmatrix} -2/4 \\ 1 \\ 8 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_2/4} \begin{bmatrix} 1 & -6/4 & 0 \\ 0 & 1 & 1/4 \\ 0 & -4 & 2 \end{bmatrix}\begin{bmatrix} -2/4 \\ 1/4 \\ 8 \end{bmatrix}$$

10

$$\xrightarrow{R_3 \leftrightarrow R_3 + 4R_2} \begin{bmatrix} 1 & -6/4 & 0 \\ 0 & 1 & 1/4 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} -2/4 \\ 1/4 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -6/4 & 0 \\ 0 & 1 & 1/4 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} -2/4 \\ 1/4 \\ 12 \end{bmatrix} \xrightarrow{R_3 \leftrightarrow R_3/3} \begin{bmatrix} 1 & -6/4 & 0 \\ 0 & 1 & 1/4 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} -2/4 \\ 1/4 \\ 4 \end{bmatrix}$$

10

$x_3 = 4$

$x_2 + x_3/4 = 1/4 \qquad x_2 + 4/4 = 1/4 \qquad x_2 = 1/4 - 1 \qquad x_2 = -3/4$ 15

$x_1 - (6/4)x_2 = -2/4 \qquad x_1 - (6/4)(-3/4) = -2/4 \qquad x_1 = -2/4 - 18/4 = -5$